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Thermal Modeling of the Human Body—Further Solutions of the Steady-State Heat Equation

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The thermal behavior of the human body has been modeled with particular emphasis on direct cooling of the skin by cooling tubes used in current extravehicular activity (EVA) space units. Steady-state analytical solutions have been obtained for several boundary conditions and for various values of the parameters involved. Although the results provide insight into the problem and even compare acceptably with some of the scant previous investigations, much more work is needed both analytically and, especially, experimentally before the numerical results can be considered reliable for biological purposes. The analysis presented herein is, nevertheless, applicable to any steady-state heat conduction problem in rectangular bodies with internal heat generation.

Nomenclature‡

a	= half-distance between cooling tubes (L)
A	= total skin surface area (L^2)
b,b_1,b_2	= depth of tissue (L)
F,F_1	= uniform heat flux $[q/(t-L^2)]$
$f(x)$	= variable heat flux $[q/(t-L^2)]$
f_a	= average heat flux, defined by Eq. (7), $[q/(t-L^2)]$
f_1,f_2	= constant heat flux, at $x=0$ and $x=\beta a$, respectively, $[q/(t-L^2)]$
k,k_1,k_2	= thermal conductivity $[q/(t-L-T)]$
n	= summation integer
Q	= defined by Eq. (2), (T/L^2)
Q_0,Q_1,Q_2	= internal heat generation rate per unit volume $[q/(t-L^3)]$
T	= tissue temperature (T)
T_1,T_2*	= constant temperature at inner core (T)
T_{ref}	= reference temperature (T)
x	= direction normal to cooling strips along skin surface (L)
y,y_1,y_2	= direction normal to skin surface (L)
α_n	= coefficient in a Fourier series, defined by Eq. (8), $[q/(t-L)]$
β	= fraction of skin cooled
λ_n	= separation coefficient, defined by Eq. (6), (L^{-1})
ξ	= integration variable (L)
ϕ	= defined by Eq. (4), (T)

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‡ The units in parentheses are L = length, p = force, q = heat (P-L), t = time, T = temperature.

Introduction

Although the thermal behavior of a biological system such as the human body is very involved, it becomes necessary in certain engineering applications to establish a model that can simulate at least certain major aspects of the heat transfer within the living tissues. Our interest in these phenomena arose from the problems of developing advanced thermoregulatory systems for protective suits to be used in hostile environments, such as outer space. These thermoregulatory systems must remove the metabolic heat generated by the body and they must compensate for all external heat effects under all conditions encountered.[1] The most recent Apollo space suits achieved metabolic heat removal by a network of water-cooled tubes in direct contact with the skin. Buchberg and Harrah[2] applied a numerical method to a steady-state model which represented the human body as a set of rectangular strips. The width of these strips was the mean half-distance between cooling tubes. The depth was divided into four layers: an outer layer of skin, a so-called functional periphery, a musculature, and an essentially constant temperature inner core. The tissues were considered isotropic. Thermal conductivity and internal heat generation were considered functions of temperature and were adjusted to compensate for the effects of blood flow.

The purpose of our analytical investigations was to obtain steady-state solutions for a similar model which, however, does not require numerical solutions. To achieve this goal, the thermal properties and the internal heat generation rate were taken as constants; and the two "working" layers of

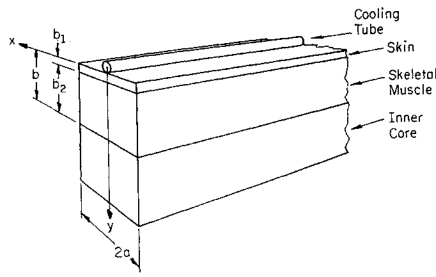

Fig. 1 Representative strip of the rectangular biothermal model.

Ref. 2, the functional periphery and the musculature, were combined into a single layer of skeletal muscle.

A secondary, more abstract purpose of this work was to obtain further solutions to the steady-state heat equation in two dimensions. These are applicable to any problem concerned with the cooling or heating of rectangular bodies with internal heat generation by strips at the surface.

Shitzer and Chato[3] recently developed a set of solutions for a modified heat equation including heat transport by blood flow. The reduction of these results to the limiting case where blood perfusion rate approaches zero is not, however, straightforward, but involves fairly complicated limit calculations. Since the solutions of the two-dimensional, steady-state heat equation with internal heat generation (Poisson's equation) are generally applicable to common engineering problems and the corresponding calculations are much simpler, we are presenting here a series of solutions for this equation. However, the treatment still emphasizes the application to conduction heat transfer in the living tissue modeled as a rectangular strip.

Analysis

Assume that each tissue is isotropic and that the thermal properties and the internal heat generation rate per unit volume are constant at values corresponding to the specific situation considered. The governing steady-state equation is

$$\nabla^2 T = -Q \tag{1}$$

where

$$Q \equiv Q_0/k \tag{2}$$

The human body, in the vicinity of the parallel cooling strips in contact with the skin, will be considered as a parallelepiped with rectangular cross section. This assumption can be justified because the cooling tubes generally run close to each other in a parallel fashion. The thermal gradient parallel to the tubes will be considered zero. The thermal field perpendicular to the tubes will be taken as symmetrical. Thus, the problem becomes two-dimensional in rectangular coordinates. The lines of symmetry, located at the center of the cooling tubes ($x = 0$) and halfway between the tubes ($x = a$), are adiabatic boundaries. Since essentially all cooling is to be achieved by the tubes, the uncontacted skin surface can also be considered as adiabatic. On the portion of the skin surface which is in contact with the cooling tubes, heat flux or temperature distributions or, possibly, convective heat transfer may be specified. Among these alternatives, the specification of the heat flux was found to be the most appropriate, versatile, and, fortunately, easiest to handle. Since the temperature of the core of the human body can be considered essentially constant, the inner boundary of the tissue can be located at the interface between the core and the skeletal muscle. Here, the appropriate boundary conditions are either constant temperature or constant heat flux.

It is clear that the actual physiological situation must be simplified considerably if analytical solutions are sought. Such simplification is necessary not only because of the mathematical difficulties involved, but also because only limited data on the parameters involved are available.[4,5]

For our purposes, a physiological model somewhat similar to that of Buchberg and Harrah[2] was considered. The inner core of the body, consisting of the internal organs and bones, was assumed to have nearly constant metabolic heat generation rate. Only the heart muscle was considered to develop varying heat generation rates, depending on the exercise rate. Since the temperature of the core is regulated closely by the body, its inclusion in the thermal model was unnecessary. Instead, the boundary condition at the interface between the inner core and the skeletal muscle was specified either as a constant temperature equal to that of the core or as a constant flux corresponding to the metabolic heat generation in the core.

The skin and the skeletal muscle were considered both as a single unit and separately. In the first case, the metabolic heat in excess of that generated within the core was assumed to be uniformly distributed in the combined tissue. In the second case, the same excess heat was assumed to be uniformly distributed only in the skeletal muscle.

Solutions in Rectangular Coordinates

The basic geometry in all cases is a rectangular surface with the x direction parallel to and the y direction normal into the skin, as shown in Fig. 1.

Case 1

Single zone (skin and skeletal muscle combined), $0 \leq x \leq a$, $0 \leq y \leq b$; variable flux over portion of the skin surface, zero flux over the rest of the skin; constant temperature at the inner boundary. The boundary conditions are

$$\text{at } x = 0, \quad \partial T/\partial x = 0 \tag{3a}$$

Table 1 Physical and physiological properties of the biothermal model corresponding to a 139 lb (63 kg) adult male with 90 mm Hg mean arterial blood pressure and total metabolic rate of 290 Btu/hr (85 w)[a]

	Property								
							Percent of total		
Region	Mass, lb	Volume, ft³	Depth, ft	Width of strip, ft	Blood flow rate, ft³/min	Oxygen consumption, ft³/min	Blood flow rate	Oxygen consumption	Thermal conductivity, Btu/hr-ft-°F
Skin	7.94	0.125	0.00812	0.064	0.0165	0.000429	8.6	4.8	0.242
Skeletal muscle	68.40	1.	0.06500	0.064	0.0300	0.001785	15.6	20	0.311
Inner core									
Heart muscle	0.66	0.010	...	...	0.1375	0.001035	4.7	11.6	...
Rest of body	62	0.965	0.06350	0.064	0.0089	0.005675	71.1	63.6	...
Whole body	139	2.100	0.13662	...	0.1929	0.008924	100	100	...

[a] Reproduced in parts from Buchberg and Harrah[2] and Ganong.[4]

$$\text{at } x = a, \quad \partial T/\partial x = 0 \tag{3b}$$

$$\text{at } y = 0, \quad \begin{Bmatrix} k(\partial T/\partial y) = f(x), 0 \le x < \beta a \\ (\partial T/\partial y) = 0, \quad \beta a < x \le a \end{Bmatrix} \tag{3c}$$

$$\text{at } y = b, \quad T = T_1 \tag{3d}$$

where $f(x)$ is a specified heat flux function in the outward direction. The solution is obtained by the transformation

$$T(x, y) = \phi(x, y) - (Q/2)y^2 + (\beta f_a/k)y \tag{4}$$

which yields

$$T = T_1 + \frac{Q}{2}(b^2 - y^2) - \frac{\beta f_a}{k}(b - y) + \frac{2}{\pi k}\sum_{n=1}^{\infty}\frac{\alpha_n}{n}[\sinh\lambda_n y - \tanh\lambda_n b \cosh\lambda_n y]\cos\lambda_n x \tag{5}$$

where

$$\lambda_n = n\pi/a \tag{6}$$

$$f_a = \frac{1}{\beta a}\int_0^{\beta a} f(\xi)d\xi \tag{7}$$

$$\alpha_n = \int_0^{\beta a} f(\xi)\cos(\lambda_n \xi)d\xi \tag{8}$$

Case 2

Single zone, $0 \le x \le a$, $0 \le y \le b$; variable flux over portion of the skin surface, zero flux over the rest of the skin; constant flux at the inner boundary. The first three boundary conditions are the same as in Case 1, Eqs. (3a–3c). The last one is replaced by

$$\text{at } y = b, \quad k(\partial T/\partial y) = F_1 = \beta f_a - Q_0 b \tag{9}$$

where the heat balance determines F_1. The temperature distribution is

$$T = T_{\text{ref}} + \frac{\beta f_a}{k}y - \frac{Q}{2}y^2 + \frac{2}{\pi k}\sum_{n=1}^{\infty}\frac{\alpha_n}{n}[\sinh\lambda_n y - \text{ctanh}\lambda_n b \cosh\lambda_n y]\cos\lambda_n x \tag{10}$$

Case 3

Two separated zones: one for the skin, $0 \le x \le a$, $0 \le y_1 \le b_1$; and one for the skeletal muscle, $0 \le x \le a$, $0 \le y_2 \le b_2$. The interface between these zones is at $y_1 = y_2 = 0$. There is a variable flux over portion of the outer surface of the skin, at $y_1 = b_1$, and zero flux over the rest of the skin. At the inner boundary of the skeletal muscle, $y_1 = b_2$, the temperature is constant. No heat generation in the skin.

Since no internal heat is assumed to be generated in the skin, $Q_1 = 0$ and Eq. (1) becomes Laplace's equation for this zone. The first two boundary conditions for both zones are the same as in Case 1. The additional boundary conditions become for the skin

$$\text{at } y_1 = b_1, \quad \begin{Bmatrix} k_1(\partial T_1/\partial y_1) = -f(x), \quad 0 \le x < \beta a \\ (\partial T_1/\partial y_1) = 0, \quad \beta a < x \le a \end{Bmatrix} \tag{11}$$

for the skeletal muscle

$$\text{at } y_2 = b_2, \quad T_2 = T_2^* \tag{12}$$

In addition, the following matching conditions must be satisfied at the interface:

$$\text{at } y_1 = y_2 = 0, \quad \begin{Bmatrix} T_1 = T_2 & (13a) \\ k_1(\partial T_1/\partial y_1) = -k_2(\partial T_2/\partial y_2) & (13b) \end{Bmatrix}$$

The transformation used was Eq. (4). The resulting temperature distributions are for the skin

$$T_1 = T_2^* + \frac{Q_2}{2}b_2^2 - \beta f_a\left[\frac{y_1}{k_1} + \frac{b_2}{k_2}\right] + \frac{2}{a}\sum_{n=1}^{\infty}\frac{\alpha_n}{\lambda_n}\left\{\frac{\tanh\lambda_n b_2}{k_2\cosh\lambda_n b_1 + k_1\tanh\lambda_n b_2 \sinh\lambda_n b_1} \times [\cosh\lambda_n y_1 - \tanh\lambda_n b_1 \sinh\lambda_n y_1] + \frac{\sinh\lambda_n y_1}{k_1\cosh\lambda_n b_1}\right\} \times \cos\lambda_n x \tag{14}$$

for the skeletal muscle

$$T_2 = T_2^* + \frac{Q_2}{2}(b_2^2 - y_2^2) - \frac{\beta f_a}{k_2}(b_2 - y_2) + \frac{2}{a}\sum_{n=1}^{\infty}\frac{\alpha_n}{\lambda_n}\left\{\frac{\tanh\lambda_n b_2}{k_2\cosh\lambda_n b_1 + k_1\tanh\lambda_n b_2\sinh\lambda_n b_1} \times [\cosh\lambda_n y_2 - \text{ctanh}\lambda_n b_2 \sinh\lambda_n y_2]\right\}\cos\lambda_n x \tag{15}$$

where λ_n, f_a, and α_n were defined by Eqs. (6–8), respectively.

Solutions for the one-dimensional cases, i.e., uniform cooling at the skin, may be readily obtained from Eqs. (5, 10, 14, and 15). This can be done by eliminating the x-dependent series and replacing the term βf_a by F. In addition, T_{ref} in Eq. (10) becomes equal to the temperature of the uniformly cooled skin.

Discussion

In order to allow comparisons with Buchberg and Harrah,[2] the numerical work illustrated by the figures was based on data given in Table 1. Heat fluxes at the skin surface either were constant or varied linearly from f_1 at $x = 0$ to f_2 at $x = \beta a$. Magnitudes of the fluxes corresponded to the metabolic heat generation rates; that is, all heat developed inside the body was removed at the skin surface. Constant temperature at the inner core was taken as 37.6°C (99.7°F). Constant flux at the inner core was taken to correspond to the metabolic heat generated within the core.

Figure 2 shows the temperature distributions at the surface of the partially cooled skin with two different types of flux

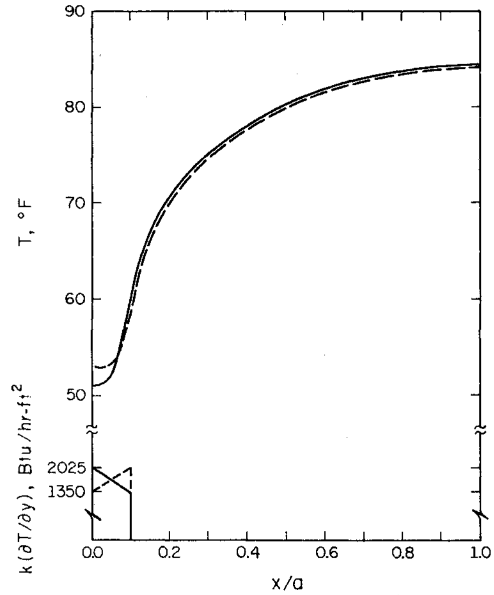

Fig. 2 Effect of reversing the specified flux function on skin temperature for Case 1 with a total metabolic rate of 2600 Btu/hr (760 w) and $\beta = 0.1$.

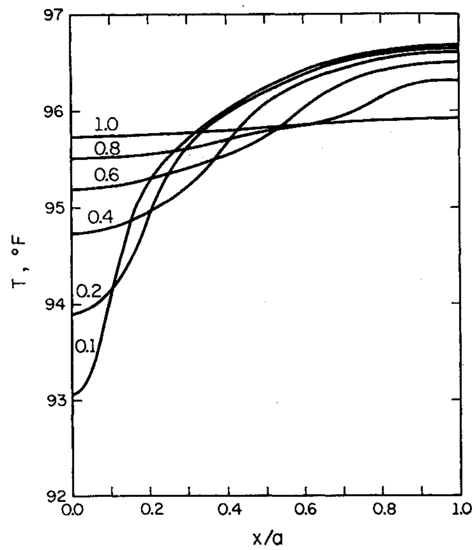

Fig. 3 Variations of skin temperature with β for Case 1 with f_1/f_2 = 1.5 and a total metabolic heat rate of 290 Btu/hr (85 w).

distribution for Case 1 at a relatively high metabolic rate of 760 w (2600 Btu/hr). The ratio of f_1/f_2 or f_2/f_1 was 1.5 and $\beta = 0.1$. The curves indicate that the shape of the flux distribution has only minor effects on the temperature distributions, which become essentially indistinguishable at a very small distance from the cooled portion of the surface.

Figure 3 shows the effect on the skin temperature of increasing the relative width of the cooled surfaces for Case 1 at

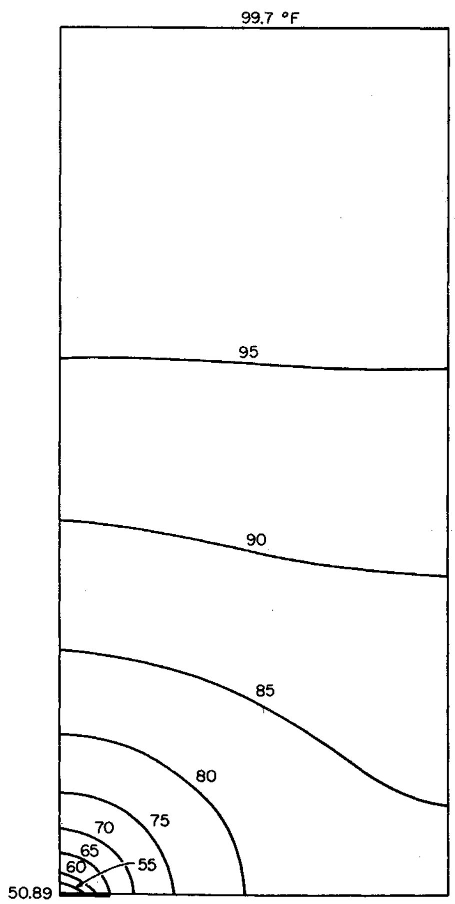

Fig. 4 Temperature distribution in the tissue for Case 1 with a total metabolic heat rate of 2600 Btu/hr (760 w) and β = 0.1.

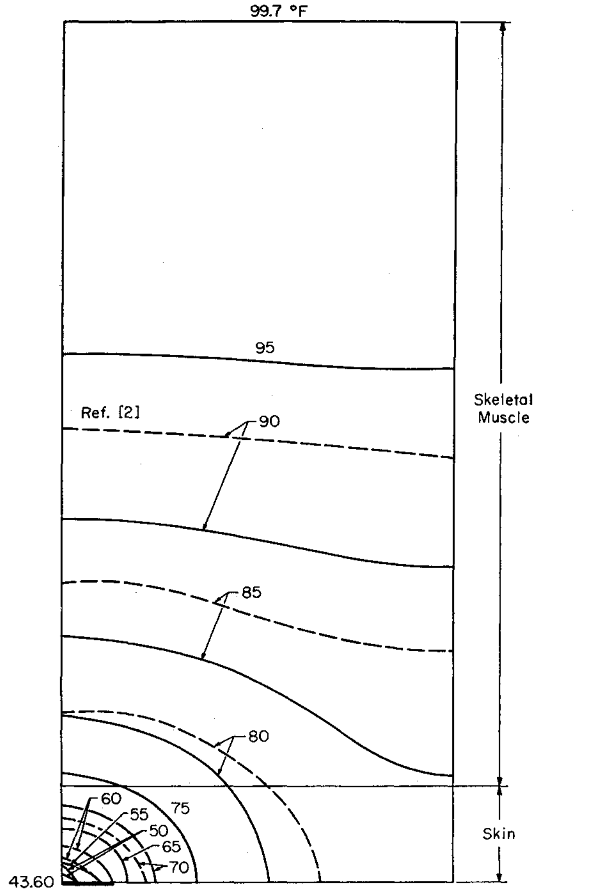

Fig. 5 Comparison of temperature distributions for Case 3 and Ref. 2 with separated zones for a total metabolic heat rate of 2600 Btu/hr (760 w) and β = 0.1.

a relatively low metabolic rate of 85 w (290 Btu/hr). As β approaches unity, the mean surface temperature of the skin approaches that of the uniformly cooled skin. The slight variation of temperature at $\beta = 1.0$ is due to the nonuniformity of the assumed heat flux ($f_1/f_2 = 1.5$).

Figure 4 shows the temperature distribution for Case 1 at a high metabolic rate. It is to be noted that for $y/a > \sim 2$, the isotherms become essentially parallel near the inner boundary; therefore, the constant temperature boundary condition at the inner core produces the same temperature distribution as a corresponding constant flux boundary condition. Because of this coincidence of results, no separate graphs are presented for Case 2. From a physiological viewpoint, it is noteworthy that the required minimum skin temperature for the removal of 2600 Btu/hr is indicated as ~51°F, a very low value from a comfort standpoint.

Figure 5 shows the results for Case 3, separated zones, together with the isotherms given for a similar case in Ref. 2. Considering the major differences in the calculation procedures, the results are remarkably close. Note that the minimum temperature for this case is 43.6°F, even lower (and, consequently, more uncomfortable) than in Fig. 4.

Since in Case 2 only fluxes were specified, the temperature level remained arbitrary as suggested by the presence of T_{ref} in Eq. (10). As noted previously, for $y/a > 2$, the constant temperature and constant flux conditions at the inner boundary produce essentially the same temperature fields. Therefore, T_{ref} should be defined to produce identical results with Case 1 for $y/a > 2$. Consequently,

$$T_{ref} = T_1 + (Q/2)b^2 - (\beta f_a/k)b \tag{16}$$

Conclusions

Perhaps the most important conclusion is that considerably more, accurate physiological data will be required to render the numerical results more meaningful from a physiological standpoint.

The previous treatment presents an approximate, steady-state model of a human body cooled by parallel cooling tubes. The analysis illustrates that the shape of the flux distribution curve at the skin has negligible effect on the temperature distribution within the tissue. It also indicates that for high metabolic rates the required minimum temperatures may become too low for comfort if the cooling strips are too narrow. If the cooling tubes are spaced too widely apart, the assumption of no heat flux at skin surface between adjacent tubes may become invalid.

Further extension of this analysis should be the exploration of transient phenomena associated with the problem. Previous work by Chato and Hertig[6] indicates that the thermal response time of the human body is relatively slow. Consequently, the steady-state solutions have only limited value if the thermal conditions, such as metabolic heat production, change frequently. This problem is under consideration by the authors.

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Spectra and Temperature of Propellant Flames during Depressurization

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A rapid-scanning spectrometer has been used to obtain emission spectra from composite propellant flames subjected to depressurization by use of rarefaction waves. A comparison of the transient-flame spectra in the 1.7 to 4.8 μ region to the spectra from steady-state flames indicates that, during the initial, rapid pressure drop, the ammonium perchlorate gasification rate was increased relative to the fuel-binder decomposition. Later in the transient, spectra characteristic of a more fuel-rich flame were observed. Transient flame temperature measurements were also made. The spectra and temperature data indicate that very severe disturbances of the combustion process can be tolerated without producing extinguishment. Data are presented which indicate that the disturbances due to changing pressure at low-pressure levels are more effective in producing extinguishment.

Introduction

A CONSIDERATION of the processes involved during extinguishment of a composite propellant by depressurization indicates that this phenomenon is one of unusual complexity. Attempts at experimental investigation or description by analytical methods are likely to meet with great difficulties. One must consider transient phenomena when the steady-state processes themselves are poorly understood; and one must cope with the essential nonlinearity of the problem where extinguishment is achieved by large excursions in pressure. To be effective, these variations must occur in a few milliseconds. Propellant heterogeneity also enters the picture. Finally, it is even difficult to find an unambiguous experimental or theoretical criterion for completion of the processes (extinguishment). However, in spite of such difficulties, some experimental characterization of extinguishment has been possible and some success has been achieved in describing gross results by use of simple models.

Most experimenters[1–7] with ammonium perchlorate propellants have followed Ciepluch's[1] basic approach and have used small motors fitted with variable area or double nozzles. In such devices the depressurization rates are affected by the transient burning of the propellant. In other tests, burning strands were extinguished by imposing pressure decays, as by use of a rarefaction tube.[8–11] One of the defects of both approaches is that only partial information is obtained from each test. Either the flame is quenched or it persists; and

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